

# $SU(3)_F$ Gauge Family Model and New Symmetry Breaking Scale From FCNC Processes

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## Abstract

Based on the  $SU(3)_F$  gauge family symmetry model which was proposed to explain the observed mass and mixing pattern of neutrinos, we investigate the symmetry breaking, the mixing pattern in quark and lepton sectors, and the contribution of the new gauge bosons to some flavour changing neutral currents (FCNC) processes at low energy. With the current data of the mass differences in the neutral pseudo-scalar  $P^0 - \bar{P}^0$  systems, we find that the  $SU(3)_F$  symmetry breaking scale can be as low as 300TeV and the mass of the lightest gauge boson be about 100TeV. Other FCNC processes, such as the lepton flavour number violation process  $\mu^- \rightarrow e^- e^+ e^-$  and the semi-leptonic rare decay  $K \rightarrow \pi \bar{\nu} \nu$ , contain contributions via the new gauge bosons exchanging. With the constrains got from  $P^0 - \bar{P}^0$  system, we estimate that the contribution of the new physics is around  $10^{-16}$ , far below the current experimental bounds.

*Keywords:* Gauge family symmetry, New symmetry breaking scale, Tri-bimaximal mixing, FCNC

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## 1. Introduction

The last five decades have witnessed the great triumph of the standard model (SM). Especially the Higgs boson was finally discovered at the Large Hadron Collider (LHC) [1, 2]. However, there are some solid experimental evidences hinting new physics beyond SM. These evidences include neutrino oscillations [3, 4], dark matter (DM) [5, 6] and baryon asymmetry of the universe (BAU) [7, 8]. Neutrino oscillations can be explained by nonzero but tiny masses of neutrinos. And the observed nearly tri-bimaximal mixing pattern [9–14] strongly indicates new symmetries, discrete or continuous, in the neutrino

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flavour sector. In general, models [15–23] inhabited by these new flavour symmetries contain new heavy particles and new CP violation (CPV) phases. As a bonus, these models may provide candidates of the DM, and new CPV sources accounting for BAU. So the flavour symmetry can be a possible solution to the puzzles mentioned above.

In SM, before electroweak symmetry is spontaneously broken, quarks and leptons are all massless. Due to the universality of gauge interactions, no quantum number can distinguish the three families. Only the Yukawa interactions can tell them apart. Thus a simple extension to SM is to introduce a new flavour symmetry among the three families, which is then broken spontaneously. In this work we take the  $SU(3)$  as the flavour symmetry group, denoted as  $SU(3)_F$ . The flavour structure of Minimal Flavour Violation in quark and lepton sectors based on family symmetries have been discussed in [24–28]. Models based on other family symmetry, such as  $SO(3)_F$  symmetry, have been discussed in [16, 17, 29–32].

In the  $SU(3)_F$  gauged family symmetry model [18], there are new interactions among the three families. The extended gauge symmetry group becomes  $SU(3)_F \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ . As the SM Higgs field being singlet under this new family symmetry transformation, new Higgs fields are needed to break the  $SU(3)_F$  symmetry. A Hermitian field  $\Phi = \Phi^\dagger$  which is adjoint representation of the  $SU(3)_F$  can do this job. Actually, to explain the mass and the mixing pattern both in quark and lepton sectors, we need two Hermitian fields  $\Phi_{1,2} = \Phi_{1,2}^\dagger$ . In the lepton sector, we also need right handed neutrinos  $N_R$  and seesaw mechanism [33–35] to explain the tiny neutrino masses. So there should be a complex symmetric Higgs  $\Phi_\nu = \Phi_\nu^T$  to generate Majorana mass terms for  $N_R$ . The new Higgs fields transform under the  $SU(3)_F$  gauge transformation as

$$\Phi_{1,2} \rightarrow g\Phi_{1,2}g^\dagger, \quad \Phi_\nu \rightarrow g\Phi_\nu g^T, \quad g(x) \in SU(3)_F. \quad (1)$$

For the representation of  $SU(3)$ , one has  $\underline{3} \otimes \underline{3} = \underline{6} \oplus \bar{\underline{3}}$  where the  $\underline{6}$  representation denoted as  $(2, 0)$  in  $p - q$  notation is symmetric while  $\bar{\underline{3}}$  is anti-symmetric. Here the  $\Phi_\nu$  is the symmetric  $\underline{6}$  representation of  $SU(3)_F$ . Seesaw mechanism can also be used to explain the mass hierarchy structures in quark and charged lepton sectors. There could also be new heavy charged fermion fields as cousins of  $N_R$ , and a new  $SU(3)_F$  singlet Higgs  $\phi_s$  to couple these new heavy fields with SM fields together. We can write down the general particle contents based on  $SU(3)_F$  gauge family symmetry with features mentioned above, as listed in Table.1. For the new gauge transformation acting in the same way on the left handed and right handed parts of all fermions, no chiral anomaly occurs here.

The general form of the Lagrangian is

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_k + \mathcal{L}_H + \mathcal{L}_Y + \mathcal{L}_n, \quad (2)$$

where  $\mathcal{L}_G$  contains the kinetic and self-interaction terms of gauge bosons, including the new gauge bosons.  $\mathcal{L}_k$  is the covariant kinetic term of the SM fermions, and contains the new gauge interactions among the three families's fermions mediated by the eight new gauge bosons. And  $\mathcal{L}_H = \mathcal{L}_{DH} - V$ , with  $\mathcal{L}_{DH}$

	Fields	Representation
SM fermions	$\begin{pmatrix} u, c, t \\ d, s, b \end{pmatrix}_L$	$(3_F, 3_C, 2_L, (1/6)_Y)$
	$(u, c, t)_R$	$(3_F, 3_C, 1_L, (2/3)_Y)$
	$(d, s, b)_R$	$(3_F, 3_C, 1_L, (-1/3)_Y)$
	$\begin{pmatrix} e, \mu, \tau \\ \nu_e, \nu_\mu, \nu_\tau \end{pmatrix}_L$	$(3_F, 1_C, 2_L, (-1/2)_Y)$
	$(e, \mu, \tau)_R$	$(3_F, 1_C, 1_L, (-1)_Y)$
SM Higgs	$H$	$(1_F, 1_C, 2_L, (1/2)_Y)$
New fermions	$U$	$(3_F, 3_C, 1_L, (2/3)_Y)$
	$D$	$(3_F, 3_C, 1_L, (-1/3)_Y)$
	$E$	$(3_F, 1_C, 1_L, (-1)_Y)$
	$N_R$	$(3_F, 1_C, 1_L, 0_Y)$
New Higgs	$\Phi_1, \Phi_2$	$(8_F, 1_C, 1_L, 0_Y)$
	$\Phi_\nu$	$(6_F, 1_C, 1_L, 0_Y)$
	$\phi_s$	$(1_F, 1_C, 1_L, 0_Y)$

Table 1: The particle contents of the model with  $SU(3)_F$  gauge symmetry and their representation of gauge group  $SU(3)_F \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ . The  $1_F(1_C, 1_L)$  means that the field is singlet of  $SU(3)_F(SU(3)_C, SU(2)_L)$  while the  $0_Y$  means the hypercharge of the field is 0. The  $\Phi_{1,2}$  are the Hermitian adjoint representation and the  $\Phi_\nu$  is the symmetric  $\bar{6}$  representation of  $SU(3)_F$ .  $\Phi_\nu$ 's VEVs produce Majorana mass terms for right handed neutrinos.  $N_R, E, D, U$  are additional heavy fields that generate mass hierarchy structures in lepton and quark sectors.

the Higgs fields' covariant kinetic terms, and  $V$  the Higgs potential.  $\mathcal{L}_{DH}$  gives masses to all the gauge bosons after spontaneously symmetry breaking(SSB).  $V$  undergoes the SSB and gives mass terms of Higgs bosons.  $\mathcal{L}_Y$  is the Yukawa interactions among all the fermions and Higgs fields. It generates masses for SM fermions and the new heavy fermions. The new fermions' kinetic and gauge interactions terms are collected in  $\mathcal{L}_n$ . Explicit expressions of these terms are listed in Appendix A.

With the eight new gauge bosons, there are tree level flavour changing neutral currents (FCNC), as well as processes that violate CP or lepton flavour numbers. These processes are suppressed in SM. In this work we use the experimental data of these processes, to get constraints on the breaking scale of this new  $SU(3)_F$  gauge symmetry.

We show the breaking pattern of the new family symmetry in Sec.2, and then give out the new effective Hamiltonian mediated by the new gauge bosons in Sec.3. After that the current experimental results of the neutral pseudo-scalar meson systems are used to constrain the broken scale of this family symmetry in Sec.4. Then we use these constraints to estimate new contributions to the semi-leptonic rare Kaon decay in Sec.5 and the lepton flavour number violating (LFNV) processes in Sec.6. A short conclusion is given in Sec.7.

## 2. Spontaneous Breaking of the $SU(3)_F$ family symmetry

Masses of the  $SU(3)_F$  family gauge bosons come from their interactions with the Higgs fields  $\Phi_1, \Phi_2$  and  $\Phi_\nu$ , as described by the covariant derivative terms of  $\Phi_{1,2} = \Phi_{1,2}^\dagger$  and  $\Phi_\nu = \Phi_\nu^T$  in  $\mathcal{L}_{Higgs}$ ,

$$\begin{aligned} D_\mu \Phi_{1,2} &= \partial_\mu \Phi_{1,2} - ig_F A_{F,\mu} \Phi_{1,2} + ig_F \Phi_{1,2} A_{F,\mu}^\dagger, \\ D_\mu \Phi_\nu &= \partial_\mu \Phi_\nu - ig_F A_{F,\mu} \Phi_\nu - ig_F \Phi_\nu A_{F,\mu}^T. \end{aligned} \quad (3)$$

The covariant kinetic terms are

$$\mathcal{L}_{DH} = \text{Tr} \left( (D_\mu \Phi_1)(D^\mu \Phi_1)^\dagger + (D_\mu \Phi_2)(D^\mu \Phi_2)^\dagger + (D_\mu \Phi_\nu)(D^\mu \Phi_\nu)^* \right). \quad (4)$$

We use  $\Phi_{1,2}$  to generate masses for quarks and charged leptons, for only one Hermitian  $\Phi$  cannot produce the observed mixing in quark sector. And  $\Phi_\nu$  generates neutrino masses through seesaw mechanism [35].

We assume that the vacuum expectation values (VEV) of  $\Phi_\nu$  are higher than that of  $\Phi_{1,2}$  and dominate the contribution to the new gauge bosons masses, since neutrinos are much lighter than the charged fermions. To show that, we use  $\Phi^E$ , which is a combination of  $\Phi_{1,2}, \Phi^E = [\Delta_1^E \Phi_1 + \Delta_2^E \Phi_2]/\xi^e$ , to generate charge leptons masses. The corresponding Yukawa interactions are

$$\begin{aligned} \mathcal{L}_{Yukawa} &= y_L^e \bar{L} H E + y_R^e \bar{e}_R \phi_s E + \frac{1}{2} \xi^e \bar{E} \Phi^E E \\ &\quad + y_L^\nu \bar{L} \tilde{H} N_R + \frac{1}{2} \xi^\nu \bar{N}_R \Phi_\nu N_R^c + H.c.. \end{aligned} \quad (5)$$

The nearly tri-bimaximal mixing pattern of neutrinos can be explained by a residual  $Z_2$  symmetry after SSB of  $SU(3)_F$ . The VEVs of the Higgs fields are assumed as the following forms [18]

$$\begin{aligned} \langle H \rangle &= v, \quad \langle \phi_s \rangle = v_s, \\ \langle \Phi^E \rangle &= \Lambda_e \text{diag}(v_1, v_2, v_3) \Lambda_e^\dagger, \\ \langle \Phi_\nu \rangle &= V_0 + \begin{pmatrix} V_1 & V_2 & V_2 \\ V_2 & V_2 & V_1 \\ V_2 & V_1 & V_2 \end{pmatrix} = \Lambda_\nu \text{diag}(V_1^\nu, V_2^\nu, V_3^\nu) \Lambda_\nu^T, \end{aligned} \quad (6)$$

where

$$\Lambda_\nu = U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \quad (7)$$

is the tri-bimaximal neutrino mixing matrix among three families, as a result of the residual  $Z_2$  symmetry.  $V_j$  ( $j = 0, 1, 2$ ) is the VEV of component field of  $\Phi_\nu$ , which possesses a residual  $Z_2$  symmetry. After diagonalising  $\langle \Phi_\nu \rangle$ , we get

$$\begin{aligned} V_1^\nu &= V_0 - V_1 + V_2, \\ V_2^\nu &= V_0 + V_1 + 2V_2, \\ V_3^\nu &= V_0 + V_1 - V_2. \end{aligned} \quad (8)$$

To get the mass eigenstates, diagonalising the mass matrices of neutrino and charged leptons as follows

$$U_\nu^T M_\nu U_\nu = \text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}), \quad U_e^\dagger M_e U_e = \text{diag}(m_e, m_\mu, m_\tau) \quad (9)$$

One has  $U_\nu = U_{TB}$  due to the  $Z_2$  symmetry and  $U_e \sim 1$  due to the approximate global  $U(1)$  symmetries after spontaneous symmetry breaking [18].  $U_e$  is expected to have similar hierarchy structure to Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix [36], which gives Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [37–39]  $U_{PMNS} = U_e^\dagger U_{TB}$  some deviation from  $U_{TB}$  with non-zero  $\theta_{13}$ . One can get the mass spectrum of SM charged leptons and neutrinos are

$$M_e^i \simeq \frac{y_L^e y_R^e v v_s}{\xi^e v_i}, \quad M_\nu^j \simeq \frac{(y_L^\nu v)^2}{\xi^\nu V_j^\nu}, \quad (10)$$

where the index  $i = 1, 2, 3$  stands for charged leptons mass eigenstates  $e, \mu, \tau$ . And  $j = 1, 2, 3$  stand for neutrinos mass eigenstates  $\nu_1, \nu_2, \nu_3$ . The observed neutrinos' mass hierarchy suggests  $V_0 \gg V_1, V_2$ . Since  $m_e \ll m_\mu < m_\tau$ , there should be  $v_1 \gg v_2 > v_3$ .

Taking all the Yukawa couplings to be nature and of order 1, we get their masses are

$$M_e^i \sim \frac{v v_s}{v_i}, \quad M_\nu^j \sim \frac{v^2}{V_j^\nu} \sim \frac{v^2}{V_0}. \quad (11)$$

Assuming  $M_\nu^j \sim 0.1\text{eV}$  and using  $m_e \sim 0.5\text{MeV}$  we can get  $V_0 \sim 10^{14}\text{GeV}$ ,  $v_1 \sim 10^5 v_s$ . The Yukawa couplings can be tuned to reduce all the scales. With  $\xi^e, \xi^\nu \sim 1$ , tuning  $y_L^e, y_R^e \sim 10^{-2}$  and  $y_L^\nu, y_R^\nu \sim 10^{-4}$ , we get  $v_1 \sim 10 v_s$ ,  $V_0 \sim 10^3\text{TeV}$ . With the assumption that  $v_s \sim \text{TeV}$ , there is  $|V_0| \gg v_1$ . So we can safely neglect contribution from  $\langle \Phi_{1,2} \rangle$  in Eq.(4) and only consider that from  $\langle \Phi_\nu \rangle$ . There is another benefit for this interval of  $v_s$ 's value. The Higgs field  $\phi_s$  can mixing with the SM Higgs field and be a cold dark matter candidate. Neglecting  $\langle \Phi_1 \rangle, \langle \Phi_2 \rangle$  in Eq.(4), we get

$$\mathcal{L} \supset g_F^2 \text{Tr} \left( A_F^\mu \Phi_\nu A_{F,\mu}^* \Phi_\nu^* + A_F^\mu \Phi_\nu \Phi_\nu^* A_{F,\mu}^\dagger + \Phi_\nu A_{F,\mu}^T A_F^{\mu*} \Phi_\nu^* + \Phi_\nu A_{F,\mu}^T \Phi_\nu^* A_F^{\mu\dagger} \right).$$

In the following parts of this paper, we denote  $A_{F,\mu}^a, A_{F,\mu}^a T^a$  as  $F_\mu^a, F_\mu$  for short. They can be parameterised by the Gell-Mann matrices with  $T^a = \lambda^a/2$ ,

$$F_\mu = F_\mu^a \frac{\lambda^a}{2} = \begin{pmatrix} \frac{1}{2} \left( F_3 + \frac{F_8}{\sqrt{3}} \right) & \frac{1}{2} (F_1 - iF_2) & \frac{1}{2} (F_4 - iF_5) \\ \frac{1}{2} (F_1 + iF_2) & \frac{1}{2} \left( \frac{F_8}{\sqrt{3}} - F_3 \right) & \frac{1}{2} (F_6 - iF_7) \\ \frac{1}{2} (F_4 + iF_5) & \frac{1}{2} (F_6 + iF_7) & -\frac{F_8}{\sqrt{3}} \end{pmatrix}_\mu. \quad (12)$$

The gauge family symmetry breaks down to residual  $Z_2$  symmetry with non-zero  $V_{0,1,2}$ . If  $V_0 \neq 0$  and  $V_1 = V_2 = 0$ , the  $SU(3)_F$  symmetry is broken down to  $SO(3)_F$  symmetry. Then there are 5 gauge family fields,  $F_1, F_3, F_4, F_5, F_6$

and  $F_8$ , gaining degenerate masses  $m = 2g_F V_0$ . The other 3 fields  $F_2, F_5, F_7$ , which corresponding to the unbroken  $SO(3)_F$  symmetry, remain massless. The  $SO(3)_F$  is besides broken with non-zero  $V_{1,2}$  and a  $Z_2$  symmetry is left. The masses of  $F_2, F_5, F_7$  are smaller comparing with the other five since  $V_{1,2} < V_0$ . We denote that

$$\frac{V_1}{V_0} \equiv r_1, \quad \frac{V_2 - V_1}{V_0} \equiv r_2, \quad (13)$$

and assume  $r_1$  and  $r_2$  are of same order of the Wolfenstein parameter  $\lambda \sim 0.22$ . A detailed analysis of neutrinos mass spectrum [18] shows  $r_1 \sim \lambda, r_2 \sim \mp 2\lambda$  can be used to explain the normal and inverted mass hierarchy spectrum of left handed neutrinos. We can use  $V_0, V_1, V_2$ , or equally  $V_0, r_1, r_2$  to get the mass spectrum of the new family gauge bosons. With the abbreviations

$$\mathcal{F}_5 = (F_1, F_3, F_4, F_6, F_8)^T, \quad \mathcal{F}_3 = (F_2, F_5, F_7)^T, \quad (14)$$

the mass terms can be expressed as

$$\mathcal{L}_{mass} = g_F^2 V_0^2 \mathcal{F}_5^T (\mathcal{M}_{5 \times 5}^2 + \delta \mathcal{M}_{5 \times 5}^2) \mathcal{F}_5 + g_F^2 V_0^2 \mathcal{F}_3^T (\mathcal{M}_{3 \times 3}^2) \mathcal{F}_3, \quad (15)$$

where the matrices are

$$\mathcal{M}_{5 \times 5}^2 = \begin{pmatrix} r_0 & 0 & 2r_1 & 2r_1 + 2r_2 & \frac{4r_1}{\sqrt{3}} + \frac{4r_2}{\sqrt{3}} \\ 0 & r_0 & 2r_1 + 2r_2 & -2r_1 & -\frac{2r_2}{\sqrt{3}} \\ 2r_1 & 2r_1 + 2r_2 & r_0 & 2r_1 + 2r_2 & -\frac{2r_1}{\sqrt{3}} - \frac{2r_2}{\sqrt{3}} \\ 2r_1 + 2r_2 & -2r_1 & 2r_1 + 2r_2 & r_0 + 2r_2 & -\frac{2r_1}{\sqrt{3}} \\ \frac{4r_1}{\sqrt{3}} + \frac{4r_2}{\sqrt{3}} & -\frac{2r_2}{\sqrt{3}} & -\frac{2r_1}{\sqrt{3}} - \frac{2r_2}{\sqrt{3}} & -\frac{2r_1}{\sqrt{3}} & r_0 + \frac{4r_2}{3} \end{pmatrix} \quad (16)$$

with  $r_0 \equiv 2 + 4r_1 + 2r_2$ , and

$$\delta \mathcal{M}_{5 \times 5}^2 = \delta \mathcal{M}_{5 \times 5}^2 (r_1^2, r_1 r_2, r_2^2) \sim \mathcal{O}(\lambda^2), \quad (17)$$

$$\mathcal{M}_{3 \times 3}^2 = \begin{pmatrix} 3r_1^2 + 5r_2 r_1 + 3r_2^2 & \frac{3r_1^2 + 8r_2 r_1 + 3r_2^2}{2} & \frac{r_2^2 - 3r_1^2 - 2r_2 r_1}{2} \\ \frac{3r_1^2 + 8r_2 r_1 + 3r_2^2}{2} & 3r_1^2 + 5r_2 r_1 + 3r_2^2 & \frac{3r_1^2 + 2r_2 r_1 - r_2^2}{2} \\ \frac{r_2^2 - 3r_1^2 - 2r_2 r_1}{2} & \frac{3r_1^2 + 2r_2 r_1 - r_2^2}{2} & 3r_1^2 + 2r_2 r_1 + r_2^2 \end{pmatrix}. \quad (18)$$

The matrix elements of  $\delta \mathcal{M}_{5 \times 5}^2$  and  $\mathcal{M}_{3 \times 3}^2$  are the of same order. The  $\mathcal{M}_{3 \times 3}^2$  and  $\mathcal{M}_{5 \times 5}^2$  can be diagnosed,

$$\hat{\mathcal{M}}_3^2 = u_{TB}^T \mathcal{M}_{3 \times 3}^2 u_{TB}, \quad \hat{\mathcal{M}}_5^2 = U_5^T \mathcal{M}_{5 \times 5}^2 U_5 \quad (19)$$

where  $u_{TB}$  and  $U_5$  are the mixing matrices

$$u_{TB} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \sqrt{\frac{2}{3}} \end{pmatrix}. \quad (20)$$

The analytical form of mixing matrix  $U_5$  is too complex to list here. If we take the assumption  $r_1 \sim \lambda$ , and  $r_2 \sim -2\lambda$  ( $r_2 \sim 2\lambda$ ) for normal hierarchy (inverted hierarchy), the numerical results are

$$U_5^{NH(IH)} = \begin{pmatrix} -\frac{1}{\sqrt{3}} & \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{6}} & -0.613(0.486) & 0.261(-0.456) \\ -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & 0.400(0.114) & 0.301(-0.487) \\ \frac{1}{\sqrt{3}} & \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{6}} & -0.613(0.486) & 0.261(-0.456) \\ 0 & -\frac{\sqrt{2}}{3} & 0 & 0.186(0.714) & 0.862(0.518) \\ \frac{1}{2} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & 0.230(0.066) & 0.174(0.281) \end{pmatrix}. \quad (21)$$

It's notable that although the mass eigenvalues depend on  $r_1, r_2$ , the mixing matrix  $u_{TB}$  do not, which is guaranteed by the residual  $Z_2$  symmetry. With  $\delta\mathcal{M}_{5\times 5}^2$  treated as perturbation, we get the mass eigenstates of the family gauge bosons

$$\begin{aligned} \mathcal{Z}_5 &= \text{diag}(Z_1, Z_2, Z_3, Z_4, Z_5) = U_5^T \mathcal{F}_5, \\ \mathcal{Z}_3 &= \text{diag}(Z_6, Z_7, Z_8) = u_{TB}^T \mathcal{F}_3. \end{aligned} \quad (22)$$

The masses of the five heavy gauge bosons are

$$\begin{aligned} M_1 &= 2g_F V_0, \quad M_2 = 2g_F(V_0^2 + 2V_1V_0 + V_2V_0)^{1/2}, \\ M_3 &= 2g_F(V_0^2 + 3V_2V_0)^{1/2}, \\ M_4 &= \frac{2\sqrt{3}}{3}g_F V_0^{1/2} \left( 2V_0 + 2V_1 + 4V_2 + 2\sqrt{4V_1^2 - 2V_1V_2 + 7V_2^2} \right)^{1/2}, \\ M_5 &= \frac{2\sqrt{3}}{3}g_F V_0^{1/2} \left( 2V_0 + 2V_1 + 4V_2 - 2\sqrt{4V_1^2 - 2V_1V_2 + 7V_2^2} \right)^{1/2}. \end{aligned} \quad (23)$$

And the masses of the three light gauge bosons, which are related to the  $SO(3)_F$  symmetry, are

$$M_6 = 2g_F|V_2 - V_1|, \quad M_7 = 3g_F|V_2|, \quad M_8 = g_F|2V_1 + V_2|. \quad (24)$$

### 3. Low Energy Effective Hamiltonian

In general the family eigenstates of the fermions are different from weak eigenstates. After the SSB of  $SU(3)_F$  family symmetry, the interactions between the new family gauge bosons and SM fermions are

$$\begin{aligned} \mathcal{L}_{int} \supset & g_F \left[ \bar{u}_L \gamma^\mu (U_L^{u\dagger} F_\mu U_L^u) u_L + \bar{u}_R \gamma^\mu (U_R^{u\dagger} F_\mu U_R^u) u_R \right] \\ & + g_F \left[ \bar{d}_L \gamma^\mu (U_L^{d\dagger} F_\mu U_L^d) d_L + \bar{d}_R \gamma^\mu (U_R^{d\dagger} F_\mu U_R^d) d_R \right] \\ & + g_F \left[ \bar{e}_L \gamma^\mu (U_L^{e\dagger} F_\mu U_L^e) e_L + \bar{e}_R \gamma^\mu (U_R^{e\dagger} F_\mu U_R^e) e_R \right] \\ & + g_F \bar{\nu}_L \gamma^\mu (U_L^{\nu\dagger} F_\mu U_L^\nu) \nu_L, \end{aligned} \quad (25)$$

where all the fermion triplets are weak eigenstates, and the corresponding mixing matrices are the clashes between weak eigenstates and family eigenstates. All the mass matrices of quarks and charged leptons are gained through SM Higgs  $H$  and  $\Phi_{1,2}$ , which are hermitian. Assuming all the Yukawa couplings to be real, as the situation in models with spontaneous CP violation, we get hermitian mass matrices, and the SSB of the new gauge symmetry and seesaw mechanism give out

$$\begin{aligned} U_L^u &= U_R^u = U^u, & U_L^d &= U_R^d = U^d, \\ U_L^e &= U_R^e = U_e, & U_L^\nu &= U_\nu = U_{TB}, \end{aligned} \quad (26)$$

where  $U_e, U_\nu$  are the mixing matrices in Eq.(9) and  $U^u, U^d$  are similar to  $U_e$ . The mixing matrices satisfy that

$$U_{CKM} = U^{u\dagger} U^d, \quad U_{PMNS} = U_e^\dagger U_{TB}, \quad (27)$$

Experimental measurement shows that the deviation between  $U_{MNSP}$  and  $U_{TB}$  is small. So we can take  $U_e \sim 1$  as the leading-order approximation. Hence the charged lepton mass eigenstates are coincident with the family eigenstates.

All the mixing matrices are physical and can be measured via the interactions among SM fermions and  $SU(3)_F$  gauge bosons. It's quite different from that in SM, where  $U^u, U^d$  and  $U_e, U_{TB}$  are not all observable, only their clashes  $U_{CKM}$  and  $U_{PMNS}$  hold physical meanings.

We can also assume that  $U^u, U^d$  and  $U_e$  have the same hierarchy structures as  $U_{CKM}$  and can be parameterised via Wolfenstein method [40]

$$U_{CKM} \sim \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 \rho e^{-i\delta} \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \rho e^{i\delta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \quad (28)$$

For  $U_e$ , we replace  $A, \lambda, \rho, \delta$  by  $A_e, \lambda_e, \rho_e, \delta_e$ . A detailed analysis of the allowed values of these parameters and the CP violation phases can be find in [41]. For the mixing matrix in up(down) quark sectors, we have mixing matrix  $U^u(U^d)$  with the parameters  $A, \lambda, \rho, \delta$  replaced by  $A_u, \lambda_u, \rho_u, \delta_u$  ( $A_d, \lambda_d, \rho_d, \delta_d$ ). Eq.(27) gives out the relations of the Wolfenstein parameters in  $U_{CKM}$ ,  $U^u$  and  $U^d$  as follows,

$$\begin{aligned} \lambda &\sim (\lambda_d - \lambda_u) \left(1 - \frac{\lambda_d \lambda_u}{2}\right), \\ A\lambda^2 &\sim A_d \lambda_d^2 - A_u \lambda_u^2, \\ e^{-i\delta} &\sim \frac{A_d \lambda_d^3 \rho_d e^{-i\delta_d} - A_d \lambda_d^2 \lambda_u + A_u \lambda_u^3 \rho_u (1 - e^{-i\delta_u})}{A_d \lambda_d^3 \rho_d - A_d \lambda_d^2 \lambda_u - A_u \lambda_d \lambda_u^2 + A_u \lambda_u^3 \rho_u}. \end{aligned} \quad (29)$$

It's known that the SM Dirac CP phase  $\delta$  is not enough to generate the observed BAU [42–44]. And the new Dirac CP phases  $\delta_e, \delta_u, \delta_d$  may help to solve the baryogenesis problem.



The low energy effective Hamiltonian mediated by these new family gauge bosons can be written down easily,

$$\mathcal{H}_{eff} = \frac{1}{S} \sum_{M,N} \sum_{a,b,c} \mathcal{C}(\mu) \xi_{ij,a}^M \xi_{kl,c}^N \frac{g_F^2 \mathcal{V}_{ab} \mathcal{V}_{cb}}{M_b^2} \mathcal{O}_{ij}^M \otimes \mathcal{O}_{kl}^N + h.c., \quad (30)$$

where  $i, j, k, l = 1, \dots, 3$  are the indices of fundamental representation of  $SU(3)_F$ , while  $a, b, c = 1, \dots, 8$  are the indices of adjoint representation of  $SU(3)_F$  and  $M_b$  is the mass of the corresponding gauge boson.  $M, N = \{u, d, e, \nu\}$  stand for the fermion's species.  $S$  is the symmetric factor,  $S = 2$  for  $\mathcal{O}_{ij}^M$  and  $\mathcal{O}_{kl}^N$  being the same, and  $S = 1$  for other situations.  $\mathcal{C}(\mu)$  are the Wilson coefficients. One can find the QCD corrections at one loop level are of order  $\sim 10\%$  [45], at the same order of corrections when we neglect the contributions of  $\Phi_{1,2}$  to the new gauge bosons masses. We do not consider the corrections of the Wilson coefficients in this work. The current operators are

$$\mathcal{O}_{ij}^N = \bar{N}_i \gamma_\mu N_j. \quad (31)$$

And the coefficients are

$$\xi_{ij,a}^N = [U^{N\dagger} T^a U^N]_{ij}. \quad (32)$$

Mixing matrix among  $SU(3)_F$  gauge bosons is a block diagonal matrix made up by  $U_{5 \times 5}$  and  $u_{TB}$ ,

$$\mathcal{V}_{ab} = [U_{5 \times 5} \oplus u_{TB}]_{ab}. \quad (33)$$

Quite a lot of effective operators occur. To suppress these new operators' contribution, we expect that the new energy scale  $V_0, V_1, V_2 \gg v \sim 173 \text{ GeV}$ . There are also some FCNC operators which are absent in SM at tree level. Such operators can contribute to the processes including the  $P^0$ - $\bar{P}^0$  mixing in neutral meson systems, as well as some LFNV processes and some CPV processes. These processes appear in SM at loop level through penguin diagrams and box diagrams, and are suppressed comparing with the tree level processes. The new gauge bosons can contribute to these processes at tree level directly. So we may find hints of these new gauge bosons in these interesting processes. In the following parts we will find the constraints given by these processes respectively.

#### 4. Mass difference of $P^0 - \bar{P}^0$

In neutral meson systems,  $P^0$  can mix with  $\bar{P}^0$ , where  $P^0$  refers to either  $K^0$ ,  $D^0$ ,  $B_d^0$  or  $B_s^0$ . Such mixing violates CP symmetry and has been studied widely [46–51]. We take  $K^0$ - $\bar{K}^0$  as an example. In SM,  $K^0$  and  $\bar{K}^0$  are mixed by  $\Delta S = 2$  interactions through box diagrams [52]. The measured tiny mass difference between  $K_L^0$  and  $K_S^0$  [53] puts stringent constraints on tree level FCNC beyond SM. The  $SU(3)_F$  family gauge bosons and their mixing can contribute

to this process at tree level. So the measured mass difference can give hint of the new gauge bosons' masses.

All the eight new gauge bosons can contribute to this mass difference. Noticed that  $Z_6, Z_7, Z_8$  are lighter than the other 5 gauge bosons, we may ignore the heavy ones and focus on these lighter ones. This approximation makes  $\mathcal{V} \sim u_{TB}$ . The form of  $U_5$  is not concerned here.

The mass difference between  $K^0$  and  $\bar{K}^0$  can be calculated using methods in [32, 54, 55]. The Hamiltonian can be written as  $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_2$ , with  $\mathcal{H}_0$  refers to the strong and electromagnetic interaction parts, which conserves the strange number. And  $\mathcal{H}_2$  is the weak interaction term and induces  $\Delta S = 2$  processes. The real parts of eigenvalues of  $\mathcal{H}$  are denoted as  $m_L, m_S$ . Their mass difference is

$$\Delta m = m_L - m_S = \text{Re} [\langle K^0 | H_2 | \bar{K}^0 \rangle + \langle \bar{K}^0 | H_2 | K^0 \rangle] / (2m_K) \quad (34)$$

The new low energy effective Hamiltonian responsible for  $K - \bar{K}$  mixing is

$$\mathcal{H}_K^{New} = \mathcal{C}_K (\bar{s} \gamma_\mu d) \otimes (\bar{s} \gamma^\mu d) + H.c.. \quad (35)$$

Here we treat  $\lambda_d$  as a small parameter and get the coefficient in Eq.(35) to the order of  $\lambda_d^2$ . At higher order the heavy family gauge bosons' effects should be take into consideration. The coefficient  $\mathcal{C}_K$  is

$$\mathcal{C}_K = \frac{1}{16} [F_K(V_1, V_2) + G_K(V_1, V_2) A_d \lambda_d^2] + \mathcal{O}(\lambda_d^3), \quad (36)$$

where

$$\begin{aligned} F_K(V_1, V_2) &= \frac{1}{6(V_2 - V_1)^2} + \frac{1}{3(2V_1 + V_2)^2} + \frac{1}{9V_2^2}. \\ G_K(V_1, V_2) &= \frac{1}{3(V_2 - V_1)^2} + \frac{2}{3(2V_1 + V_2)^2} - \frac{2}{9V_2^2}. \end{aligned} \quad (37)$$

The contribution of  $G_K(V_1, V_2)$  are at order of  $\lambda_d^2$ . If we assume  $\lambda$  and  $\lambda_d$  are of the same order, then the contribution of  $G_K(V_1, V_2)$  can be omitted as the contributions of the heavy gauge bosons. This approximation is equivalent to setting the mixing matrix  $U^d \sim 1$ .

To get the matrix element  $\langle \bar{K}^0 | \mathcal{O} | K^0 \rangle$ , we use the vacuum insertion approximation (VIA). The result is

$$\langle \bar{K}^0 | (\bar{s} \gamma^\mu d) \otimes (\bar{s} \gamma^\mu d) | K^0 \rangle = \frac{2}{3} N_1 + \frac{1}{3} N_2, \quad (38)$$

where [56]

$$\begin{aligned} N_1 &\equiv \langle \bar{K}^0 | \bar{s} \gamma^5 d | 0 \rangle \langle 0 | \bar{s} \gamma^5 d | K^0 \rangle, \\ N_2 &\equiv \langle \bar{K}^0 | \bar{s} \gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma^\mu \gamma_5 d | K^0 \rangle. \end{aligned} \quad (39)$$

With the definition of Kaon decay constant  $f_K$ ,

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 d | K^0(p) \rangle = i f_K p^\mu, \quad (40)$$

we get

$$N_1 = \frac{f_K^2 m_K^4}{(m_s + m_d)^2}, \quad N_2 = f_K^2 m_K^2. \quad (41)$$

To the lowest order of  $\lambda_d$ ,

$$\begin{aligned} \langle \bar{K}^0 | \mathcal{H}_2^{New} | K^0 \rangle &= \frac{F_K(V_1, V_2)}{16} \frac{f_K^2 M_K^2}{6M_K} \left[ 1 + 2 \frac{M_K^2}{(m_s + m_d)^2} \right] \\ &= \frac{F_K(V_1, V_2) f_K^2 M_K}{96} [1 + 2R(\mu)]. \end{aligned} \quad (42)$$

The hadronic matrix uncertainties will modify the relation above [45, 57]. From Eq.(34), the new family interaction contributes to the mass difference via a new term in addition to that in SM as

$$\Delta m^{New} = \frac{F_K(V_1, V_2) f_K^2 M_K}{48} [1 + 2R(\mu)]. \quad (43)$$

If the new contribution saturate the mass difference, then

$$\frac{1}{F_K(V_1, V_2)} \geq \frac{f_K^2 M_K}{48 \Delta m^{New}} [1 + 2R(\mu)] \sim \frac{f_K^2 M_K}{48 \Delta m_K}. \quad (44)$$

With Eq.(13), it's easy to get

$$V_1^2 \geq \frac{f_K^2 M_K}{864 \Delta m_K} \left[ \frac{3r_1^2}{r_2^2} + \frac{2r_1^2}{(r_1 + r_2)^2} + \frac{6r_1^2}{(3r_1 + r_2)^2} \right]. \quad (45)$$

Using the experimental data [53, 58] listed in Table.2, and taking the assumption that  $r_1 \sim \lambda$  and  $r_2 \sim 2\lambda$ , we can get the bounds of the symmetry broken scales which are about

$$V_1 \geq 69.8 TeV, \quad V_2 \geq 209 TeV, \quad V_0 \geq 317 TeV. \quad (46)$$

The lower bounds of  $V_0, V_1$  and  $V_2$  as functions of  $r_1, r_2$  are shown in Fig.1. A similar analysis can be carried out in  $D - \bar{D}$ ,  $B - \bar{B}$  and  $B_s - \bar{B}_s$  systems. The effective Hamiltonian terms at the lowest order are

$$\begin{aligned} \mathcal{H}_D^{New} &= \mathcal{C}_D (\bar{u} \gamma_\mu c) \otimes (\bar{u} \gamma^\mu c), \\ \mathcal{H}_{B_d}^{New} &= \mathcal{C}_{B_d} (\bar{b} \gamma_\mu d) \otimes (\bar{b} \gamma^\mu d), \\ \mathcal{H}_{B_s}^{New} &= \mathcal{C}_{B_s} (\bar{b} \gamma_\mu s) \otimes (\bar{b} \gamma^\mu s), \end{aligned} \quad (47)$$

where

$$\mathcal{C}_D \sim \frac{F_D(V_1, V_2)}{16}, \quad \mathcal{C}_{B_d} \sim \frac{F_{B_d}(V_1, V_2)}{16}, \quad \mathcal{C}_{B_s} \sim \frac{F_{B_s}(V_1, V_2)}{16}, \quad (48)$$

and

$$\begin{aligned} F_D(V_1, V_2) &= F_{B_d}(V_1, V_2) = F_K(V_1, V_2), \\ F_{B_s}(V_1, V_2) &= \frac{4}{3(2V_1 + V_2)^2} + \frac{1}{6(V_2 - V_1)^2}. \end{aligned} \quad (49)$$

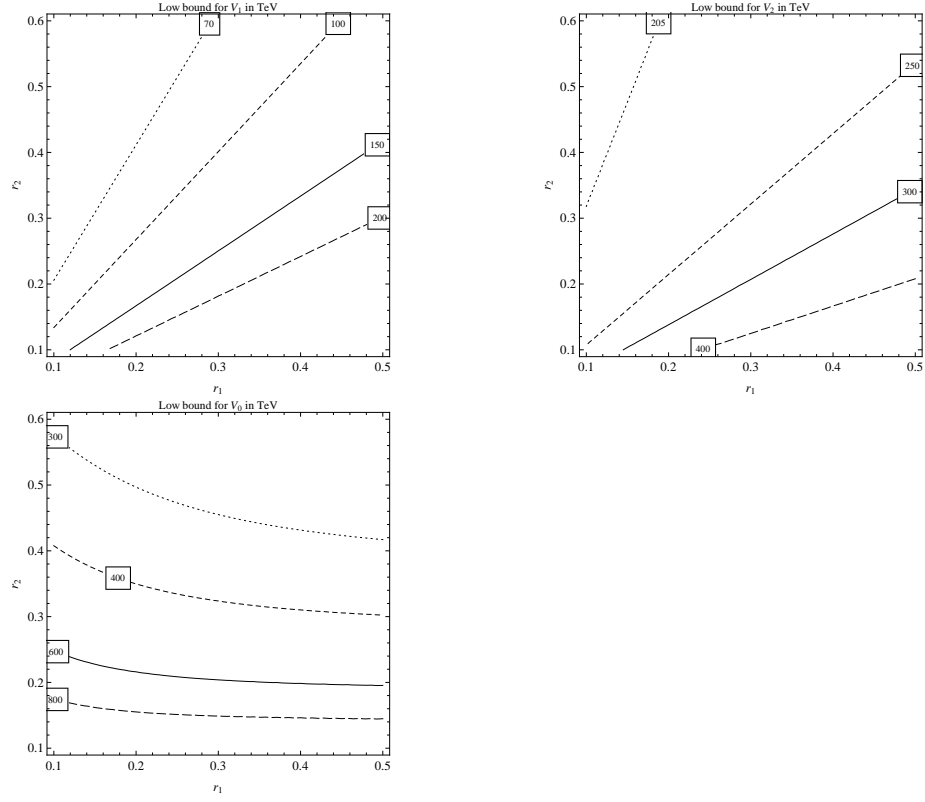


Figure 1: The lower bounds of  $SU(3)_F$  breaking scale  $V_1, V_2$  and  $V_0$  in  $TeV$  given by neutral Kaon system with different  $r_1$  and  $r_2$ .

$P^0 - \bar{P}^0$	$[\Delta m_{meson}]^{PDG}$	$M_{meson}$	$f_{meson}$	$V_1 \geq$
$K - \bar{K}$	$(3.483 \pm 0.006) \times 10^{-12}$	497.6	$156 \pm 1.2$	$7.0 \times 10^7$
$D - \bar{D}$	$(1.57^{+0.39}_{-0.41}) \times 10^{-11}$	$1864.86 \pm 0.13$	$206 \pm 11$	$8.4 \times 10^7$
$B_d - \bar{B}_d$	$(3.337 \pm 0.033) \times 10^{-10}$	$5279.58 \pm 0.17$	$195 \pm 11$	$2.9 \times 10^7$
$B_s - \bar{B}_s$	$(116.4 \pm 0.5) \times 10^{-10}$	$5366.77 \pm 0.24$	$243 \pm 11$	$0.7 \times 10^7$

Table 2: Constrains on the family symmetry breaking scale  $V_1$  from different neutral meson systems. The values are all in unit of  $MeV$ .

To the lowest order, we neglect the mixing matrices  $U^u, U^d$ , and the same mixture of  $Z_i$  in  $F_2$  and  $F_5$  lead to the result  $F_D = F_{B_s} = F_K$ . Using data from [53, 58–61] we can get other lower bounds, which are list in the Table.2.

It's obvious from Table.2 that the  $K^0-\bar{K}^0$  system and  $D^0-\bar{D}^0$  system give the most stringent constraints on  $V_1$ . The lower bounds turn out to be about  $70 \sim 84$  TeV.  $V_0$  can be got through  $V_1$  with Eq.(13), which turns out to be about 300TeV. To apply seesaw mechanism at this scale, we need tuning the Yukawa coupling to  $10^{-4}$ . Although not very nature, it's much better than the situation in SM. It is notable that the constrains on the scales are not depend on the gauge coupling strength  $g_F$ . If we take it on the same order as the weak interaction, the mass of the new lightest gauge family boson can be about 100TeV. This energy scale is at the reach of the next generation 100TeV colliders.

## 5. Semi-leptonic decay of Kaon

In SM FCNC processes occur at loop level through box diagrams and penguin diagrams [45, 62]. These processes are suppressed by high order coupling, loop factor  $1/16\pi^2$ , and CKM factors in power of  $\lambda \sim 0.22$ . With the new gauge bosons, FCNC process can happen at tree level. The new gauge bosons may manifest themselves and play a crucial roles in such processes. On the other hand, due to their heavy masses, there is almost no significant effect on the SM tree level allowed channels. For example, the rare kaon decay process  $K \rightarrow \pi\nu\bar{\nu}$ , and LFNV processes  $\mu \rightarrow eee$ .

In SM, the rare Kaon decay processes are induced by Z-penguin diagram and box diagram. And the channel  $K_L \rightarrow \pi^0\nu\bar{\nu}$  violates CP directly [63], providing same flavour contents of the final neutrino pair.

The couplings between SM fermions and the new gauge bosons provide several new  $|\Delta S| = 1$  low energy effective Hamiltonian terms, for the final neutrinos with arbitrary flavour contents, the effective Hamiltonian terms are:

$$\mathcal{H}_{eff}(K \rightarrow \pi\nu\bar{\nu}) = \mathcal{C}_{lm}(\bar{s}\gamma_\mu d) \otimes (\bar{\nu}_l\gamma^\mu\nu_m) + h.c. \quad (50)$$

where  $l, m = e, \mu, \tau$ , and the numerical values of the coefficient matrix elements for  $r_1 \sim \lambda$ ,  $r_2 \sim 2\lambda$  are

$$\mathcal{C}_{lm} = \begin{pmatrix} -0.172 & -0.616 & -0.329 \\ 0.729 & 0.329 & 1.40 \\ -0.248 & -1.54 & -0.157 \end{pmatrix}, \quad (51)$$

The diagonal matrix elements correspond to same flavour neutrino final states. We can sum these channels incoherently and get the coefficient being  $\sum_l |\zeta_l|^2 \sim 0.16$ .

We only focus on the left-handed neutrinos, thus the leptonic current takes a  $V-A$  form. As for the hadronic current, since  $\langle \pi | A^\mu | K \rangle = 0$ , the final result only depends on  $\langle \pi | V - A | K \rangle$ . We have

$$\mathcal{H}_{eff}^{CP} = \frac{0.4}{8V_0^2} (\bar{s}d)_{V-A} (\bar{\nu}_\alpha \nu_\alpha)_{V-A} + h.c., \quad (52)$$

where the neutrino pairs belong to weak eigenstates and have the same flavour. Using the isospin symmetry relation:

$$\begin{aligned} \langle \pi^+ | (\bar{s}d)_{V-A} | K^+ \rangle &= \sqrt{2} \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle, \\ \langle \pi^0 | (\bar{s}d)_{V-A} | K_L \rangle &= \sqrt{2} \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle, \end{aligned} \quad (53)$$

we have

$$\frac{Br(K_L \rightarrow \pi^0 \bar{\nu}_\alpha \nu_\alpha)|_{New}}{Br(K^+ \rightarrow \pi^0 \nu_e e^+)|_{SM}} \sim \frac{Br(K^+ \rightarrow \pi^+ \bar{\nu}_\alpha \nu_\alpha)|_{New}}{Br(K^+ \rightarrow \pi^0 \nu_e e^+)|_{SM}} \sim \left[ \frac{2 \times 0.4}{8G_F V_0^2} \right]^2, \quad (54)$$

Taking  $V_0 \sim 3 \times 10^2 TeV$  and using the result  $Br(K^+ \rightarrow \pi^0 \nu e^+) = (5.07 \pm 0.04)\%$  [53], we can get the branch ratio

$$Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})|_{New} \approx Br(K_L \rightarrow \pi^0 \nu \bar{\nu})|_{New} \simeq 4.6 \times 10^{-16}. \quad (55)$$

The SM predicts these semi-leptonic FCNC processes have tiny branch ratios [64]

$$\begin{aligned} Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})|_{SM} &= (1.5_{-1.2}^{+3.4}) \times 10^{-10}, \\ Br(K_L \rightarrow \pi^0 \nu \bar{\nu})|_{SM} &= (2.6 \pm 1.2) \times 10^{-11}. \end{aligned} \quad (56)$$

We find the contributions from new gauge bosons are far below the SM prediction in Eq.(56). The CP violation in  $K_L \rightarrow \pi \nu \nu$  is still dominated by SM contribution.

## 6. Lepton flavour changing processes

In SM, LFNV processes are caused by the non-zero masses of neutrinos [65] and neutrino mixing. There are several interesting LFNV processes, such as  $\mu^- \rightarrow e^- + \gamma$  and  $\mu^- \rightarrow e^- e^+ e^-$ . In SM, these processes are loop level effects and highly suppressed. SM predictions of these processes are hopelessly small [66],

$$\begin{aligned} Br(\mu \rightarrow e \gamma)|_{SM} &\sim 10^{-54}, \\ Br(\mu \rightarrow e e e)|_{SM} &\sim 10^{-56}. \end{aligned} \quad (57)$$

The experimental bounds on the branch ratios at 90% C.L. are [53]

$$Br(\mu^- \rightarrow e^- \gamma)|_{Exp} < 1.2 \times 10^{-11} \quad (58)$$

$$Br(\mu^- \rightarrow e^- e^+ e^-)|_{Exp} < 1.0 \times 10^{-12}. \quad (59)$$

The process  $\mu \rightarrow e\gamma$  are not influenced by the new gauge bosons at tree level. However, for  $\mu^- \rightarrow e^- e^+ e^-$ , there are tree level contributions mediated by the new gauge bosons. Here with the assumption that  $U_e \sim 1$ , we get the effective Hamiltonian for this process is

$$\mathcal{H}_{eff}(\mu \rightarrow 3e) = \frac{1}{8V_0^2} \frac{F(r_1, r_2)}{G(r_1, r_2)} (\bar{e}\gamma_\mu\mu) \otimes (\bar{e}\gamma^\mu e) + H.c., \quad (60)$$

where

$$\begin{aligned} G(r_1, r_2) = & 216r_1^3 - 72r_1^2r_2^2 + 432r_1^2r_2 + 198r_1^2 - 96r_1r_2^3 + 216r_1r_2^2 \\ & + 264r_1r_2 + 60r_1 - 24r_2^4 + 16r_2^3 + 74r_2^2 + 40r_2 + 6, \\ F(r_1, r_2) = & -12\sqrt{3}r_1^3 - 24r_1^3 + 6r_1^2r_2^2 - 21\sqrt{3}r_1^2r_2 - 51r_1^2r_2 - 7\sqrt{3}r_1^2 \\ & - 14r_1^2 + 8r_1r_2^3 - 12\sqrt{3}r_1r_2^2 - 32r_1r_2^2 - 6\sqrt{3}r_1r_2 - 22r_1r_2 \\ & - \sqrt{3}r_1 - 2r_1 + 2r_2^4 - 3\sqrt{3}r_2^3 - 5r_2^3 - \sqrt{3}r_2^2 - 8r_2^2 - 2r_2. \end{aligned} \quad (61)$$

Taking  $r_1 \sim \lambda$ ,  $r_2 \sim 2\lambda$ , we get  $F(r_1, r_2)/G(r_1, r_2) \sim -0.12$ . The branching ratio for this channel is

$$Br(\mu \rightarrow 3e) = \frac{\Gamma_{\mu \rightarrow 3e}}{\Gamma_{\mu \rightarrow e\nu\bar{\nu}}} \sim \left[ \frac{-0.12\sqrt{2}}{8G_F V_0^2} \right]^2. \quad (62)$$

Assuming  $V_0 \geq 3 \times 10^2 TeV$ , we get

$$Br(\mu \rightarrow 3e) \leq 4.1 \times 10^{-16}. \quad (63)$$

This result is much larger than the SM prediction in Eq.(57) but still below the experimental bound [53]. The contribution of new physics in this process is of same order as that in  $K_L \rightarrow \pi\nu\nu$ . Both of their initial flavours are changed. And they are induced by the mixing among the heavy family gauge bosons  $F_1, F_4, F_6$  and  $F_3, F_8$ . There are many similar processes, such as the rare B decays through  $B \rightarrow X_s \mu^- \mu^+$ , rare Kaon decay through  $K_L \rightarrow \pi^0 e^+ e^-$ ,  $K_L \rightarrow \mu^+ \mu^-$ . Their branching ratios are of the same order, i.e.  $10^{-16}$  from the new gauge bosons' contributions. And they are all below the various experimental bounds. These results make the lower bound  $V_0 \sim 300 TeV$  safe.

## 7. conclusion

We have investigated the structure of  $SU(3)_F$  gauge family symmetry model and its low energy phenomenal results in flavour physics. This family symmetry undergoes spontaneous breaking to  $SO(3)_F$  and then to a residual  $Z_2$  symmetry. Seesaw mechanism is widely used both in leptonic sector and quark sector to explain the observed mass hierarchy and mixing structure, especially the neutrinos' mass spectrum. The equality of seesaw scale and flavour symmetry breaking scale needs a tuning of the Yukawa couplings, about  $10^{-4}$ , which are much softer than SM. New scalar field is introduced and may be a dark matter

candidate. Also new CP violation phases appear and may provide a solution to the baryon asymmetry in the universe. The symmetry breaking mode makes the new gauge bosons can be divided into two groups. Their mass scales can be constrained through the mass differences of  $P^0$ - $\bar{P}^0$  meson systems. We get the broken scale of the new gauge family symmetry is about  $V_0 \geq 300$  TeV, and mass of the lightest new gauge boson can be low as 100 TeV. These new gauge bosons can induce FCNC processes at tree level, and their contributions are suppressed by their heavy masses and the resulting branching ratios are about  $10^{-16}$ , which is 4  $\sim$  5 order below the current experimental bounds. We expect the improvement of the rare FCNC processes' measurements, as well as some exotic processes' discovery, which may be found in the next running of LHC and the next generation colliders of 100 TeV, can throw some light upon this new flavour symmetry.

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### Appendix A.

The field strengths of all gauge fields, including the  $SU(3)$  family symmetry, are defined as

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_{F,\nu}^a - \partial_\nu A_{F,\mu}^a + g_F f^{abc} A_{F,\mu}^b A_{F,\nu}^c, \\ G_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c, \\ W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_w \epsilon^{abc} W_\mu^b W_\nu^c, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \tag{A.1}$$

We define the covariant derivative as

$$\begin{aligned} D_\mu &= \partial_\mu - ig_F A_{F,\mu}^a T^a - ig_s G_\mu - g_w W_\mu + ig'_w Y B_\mu \\ &= D_\mu^{SM} - ig_F A_{F,\mu}^a T^a. \end{aligned} \tag{A.2}$$

The full Lagrangian is

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_k + \mathcal{L}_H + \mathcal{L}_Y + \mathcal{L}_N, \tag{A.3}$$



with each term defined as follows

$$\mathcal{L}_G = -\frac{1}{4} (F_{\mu\nu}^a F^{a\mu\nu} + G_{\mu\nu}^b G^{b\mu\nu} + W_{\mu\nu}^c W^{c\mu\nu} + B_{\mu\nu} B^{\mu\nu}) \quad (\text{A.4})$$

$$\mathcal{L}_k = \bar{u}_{L,R} i\gamma^\mu D_\mu u_{L,R} + \bar{d}_{L,R} i\gamma^\mu D_\mu d_{L,R} + \bar{e}_{L,R} i\gamma^\mu D_\mu e_{L,R} + \bar{\nu}_L i\gamma^\mu D_\mu \nu_L, \quad (\text{A.5})$$

$$\begin{aligned} \mathcal{L}_H &= \mathcal{L}_{DH} - V[H, \Phi_1, \Phi_2, \Phi_\nu, \phi_s] \\ &= (D_\mu^{SM} H)^\dagger (D^{\mu, SM} H) + \text{Tr} (D_\mu \Phi_1 (D^\mu \Phi_1)^\dagger) + \text{Tr} (D_\mu \Phi_2 (D^\mu \Phi_2)^\dagger) \\ &\quad + \text{Tr} (D_\mu \Phi_\nu (D^\mu \Phi_\nu)^*) + \partial_\mu \phi_s \partial^\mu \phi_s - V(H, \Phi_1, \Phi_2, \Phi_\nu, \phi_s). \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} \mathcal{L}_Y &= y_L^u \bar{l} H U + y_R^u \bar{u}_R \phi_s U + \frac{1}{2} \bar{U} (\Delta_1^U \Phi_1 + \Delta_2^U \Phi_2) U \\ &\quad + y_L^d \bar{l} \tilde{H} D + y_R^d \bar{d}_R \phi_s D + \frac{1}{2} \bar{D} (\Delta_1^D \Phi_1 + \Delta_2^D \Phi_2) D \\ &\quad + y_L^e \bar{l} H E + y_R^e \bar{e}_R \phi_s E + \frac{1}{2} \bar{E} (\Delta_1^E \Phi_1 + \Delta_2^E \Phi_2) E \\ &\quad + y_L^\nu \bar{l} \tilde{H} N_R + \frac{1}{2} \xi^\nu \bar{N}_R \Phi_\nu N_R^c + H.C.. \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} \mathcal{L}_N &= i\bar{U} \gamma^\mu (\partial_\mu - ig_s G_\mu - ig_F A_{F,\mu}^a T^a) U + i\bar{D} \gamma^\mu (\partial_\mu - ig_s G_\mu - ig_F A_{F,\mu}^a T^a) D \\ &\quad + i\bar{E} \gamma^\mu (\partial_\mu - ig_F A_{F,\mu}^a T^a) E + i\bar{N}_R \gamma^\mu (\partial_\mu - ig_F A_{F,\mu}^a T^a) N_R. \end{aligned} \quad (\text{A.8})$$

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